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CN A PILOT LINEAR PROGRAMMING MODEL FOR ASSESSING PHYSICAL IMPACT ON THE ECONOMY OF A CHANGING ENERGY PICTURE

George B. Dantzig, et al

Stanford University

Prepared for:

Office of Naval Research
National Science Foundation
Atomic Energy Commission

August 1975

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BY

GEORGE B. DANTZIG and S. C. PARIKH

TECHNICAL REPORT SOL 75-14R
AUGUST 1975

Systems Optimization Laboratory

Department of Operations Research

Stanford University

Stanford California 94305

18

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SOL 75-14R	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ON A PILOT LINEAR PROGRAMMING MODEL FOR ASSESSING PHYSICAL IMPACT ON THE ECONOMY OF A CHANGING ENERGY PICTURE		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) George B. DANTZIG and S.C. PARIKH		8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0865 N00014-75-C-0267
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Operations Research, SOL Stanford University Stanford, CA 94305		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 047-143 NR 047-064
11. CONTROLLING OFFICE NAME AND ADDRESS Operations Research Program Code 434 Office of Naval Research Arlington, Virginia 22217		12. REPORT DATE August 1975 13. NUMBER OF PAGES 14
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; its distribution is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

LARGE-SCALE LINEAR PROGRAMMING ECONOMIC IMPACT

ENERGY-ECONOMY MODELLING

PHYSICAL FLOWS

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

See reverse side

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S/N 0102-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. ABSTRACT: SOL 75-14R

This paper deals with a dynamic linear programming model on a pilot scale that attempts to describe in physical terms many of the technological interactions within and across the sections of the American Economy, including a detailed energy sector. The general aim of the model is to provide information on what the country would achieve over the long run in physical terms in the face of a changing energy picture.

The model is expected to (i) incorporate most recent available data of sufficiently good quality (albeit in an aggregated form) that it can be used to generate some meaningful scenarios showing how the economy might be affected if the energy picture evolves in a specified way, and (ii) provide us at the Systems Optimization Laboratory a prototype for research in solving large-scale linear programming models of energy systems.

ON A PILOT LINEAR PROGRAMMING MODEL FOR ASSESSING PHYSICAL IMPACT ON THE ECONOMY OF A CHANGING ENERGY PICTURE†

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George B. Dantzig and S. C. Parikh‡

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DEPARTMENT OF OPERATIONS RESEARCH

Stanford University
Stanford, California

†Prepared for presentation at the SIMS Research Application Conference Energy at Alta, Utah, July 7-11, 1975.

‡University of Santa Clara, Santa Clara, California.

Research and reproduction of this report were partially supported by the Office of Naval Research under Contracts N00014-75-C-0865 and N00014-75-C-0267; the Energy Research and Development Administration Contract AT(04-3)-326 PA #18; and National Science Foundation Grant MPS71-03341 A03.

ON A PILOT LINEAR PROGRAMMING MODEL FOR ASSESSING PHYSICAL IMPACT

ON THE ECONOMY OF A CHANGING ENERGY PICTURE

by George B. Dantzig[*] and S. C. Parikh[*†]

1. Introduction. This paper reports on some of the ongoing work on models of energy systems at the Institute for Energy Studies and the Systems Optimization Laboratory of the Stanford University. It deals with a dynamic, linear programming model on a pilot scale that attempts to describe in physical terms many of the technological interactions within and across the sectors of the American Economy, including a detailed energy sector. The general aim of the model is to provide information on what the country could achieve over the long term (say 30 years) in physical terms in the face of changing energy picture.

Mathematical programming models that link activities of the economic sectors with those of a detailed energy sector and describe interactions over time can provide comprehensive and effective means for evaluating the nature and extent of the impact on the economy in general and the living standards in particular, of the realizations of various scenarios concerning the availability and the mix of raw energy and the type of conversion technology utilized. However, simple, rough calculation below shows that any such model can become quite large and perhaps unmanageable if sufficient care is not exercised in its development.

The input-output matrix provides a convenient vehicle for incorporating into a mathematical programming model the technological and many of the economic interactions of the economy. Despite its shortcomings, such as constant returns to scale, fixed technology and time delays involved in data collection and publication, it is attractive because it provides an internally consistent and a single most comprehensive data source. In its standard published form, it is available as an 87-sector matrix.[1] Next, the energy sector may be modeled

[*]Department of Operations Research, Stanford University, Stanford, California.

[†]University of Santa Clara, Santa Clara, California.

[1]A much more detailed 367-sector matrix is also available from the Department of Commerce.

by approximately 150 equations per period including the capacity constraints on activity levels of the energy processes and interperiod capacity carryover constraints. See [12], for example. An order of magnitude for the number of constraints per period in an integrated model with a reasonable level of detail is therefore computed to be 400: 87 for industrial activity, 2 x 87 for constraints of capacities on levels of industrial activity and for capacity carryover from one period to the next, and about 150 for the detailed energy sector. A 20-25 period model (e.g. a 25-year annual model, or a 75-year tri-annual model, etc.) would therefore have approximately 8000-10,000 constraints and much more if more detailed input-output matrix and energy sectors are employed. While linear programming models of this magnitude are certainly not considered to be impossible to solve, they would be among the largest models built to date. More importantly, preparation, testing/validation, and production runs for such a model would most likely consume both substantial sums of money and substantial amounts of time.

The aim of this exercise is not to suggest that one build such large models but rather to draw attention to the potential model size resulting from indiscriminate modeling and the difficulties that may arise.

It is therefore absolutely essential that a critical and scientific assessment be made of the exact nature of the formulation, scope and limitations of such a class of models. In particular, it is important to obtain answers to questions along the following lines:

Formulation--In specific terms, what aspects should be modeled (endogenous), and what aspects should be assumed (exogenous) and what information should flow between periods? What linkages between the energy sector and the economy should be formulated and how?

Availability of data--What are the data requirements of the model? Are such data available? If not, is it possible to obtain satisfactory quick-and-dirty estimates to satisfy the immediate needs? And, what types of studies are needed to develop better quality data over a longer term?

Information from the model--What type of meaningful information can the model provide? What are the different objective functions that can be evaluated? At what level of detail should the model be formulated to provide the information desired?

Computation of solutions--Can the model be (efficiently) solved on the computer? What would the computational costs be? What refinements or special purpose algorithms exist (that perhaps require further research and) that can substantially reduce the computational costs?

These and other similar considerations point towards a need for developing and experimenting with a much smaller model that incorporates many, if not all, of the essential features of its larger counterpart. Our pilot model is an attempt to satisfy such a need. We believe that it will be small enough so that when implemented on the computer, it will have the agility for extensive experimentation. On the other hand, we also expect that it will incorporate most recent available data of sufficiently good quality (albeit in an aggregated form) that it can also be used to generate some meaningful scenarios showing how the economy might be affected if the energy picture evolves in a specified way.

In what follows, we first describe the model in some detail. Next, we give a brief and general mathematical statement of the model. Finally, we briefly review the current status of the model, its mathematical structure, and possible solution approaches.

2. Description of the model. In the model, a 23-sector input-output matrix represents various industrial processes of the economy (Exhibit 1). The net output from the industry, together with net imports, meets the national bill of goods for consumption, capital formation and government services. The energy demands of the economy are met by the activities of the energy sector. The nature and extent of the capacity expansion in both the energy sector and the rest of the economy are endogenously determined. Finally, the exogenously given workforce provides the manpower necessary to sustain industrial production, energy processing and capacity expansion.

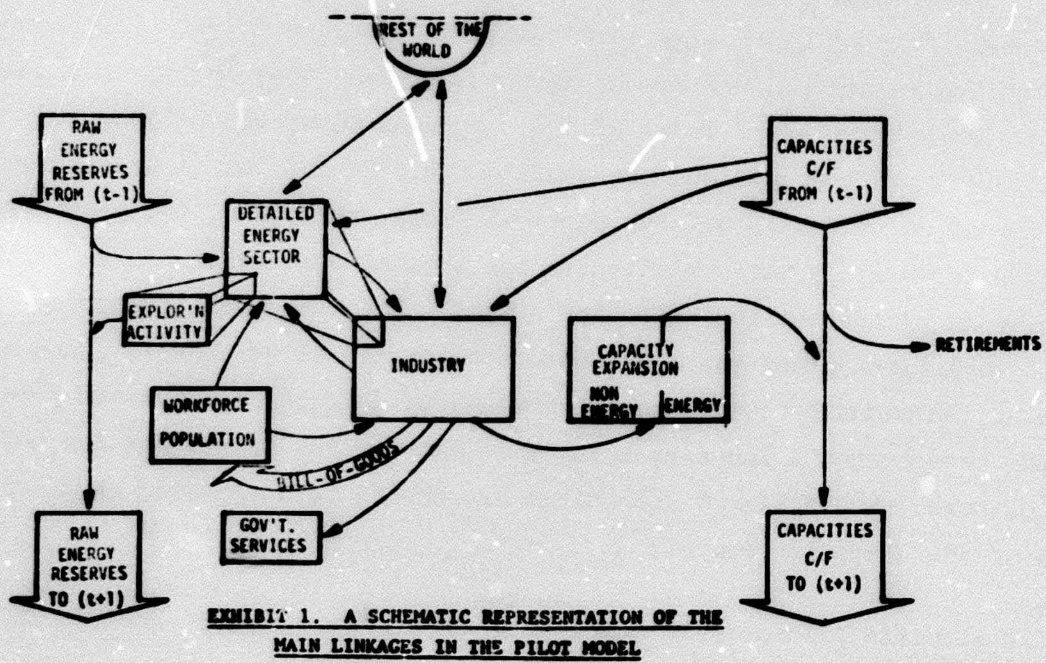

EXHIBIT 1. A SCHEMATIC REPRESENTATION OF THE MAIN LINKAGES IN THE PILOT MODEL

The detailed energy sector in the model includes technological description of the raw material extraction and energy conversion processes (Exhibits 2 and 3). Uranium mining, milling, conversion, enrichment and fabrication, light water reactor, fast breeder reactor, and spent fuel reprocessor are among the nuclear fuel based processes in the model. Oil and gas exploration and production, oil refining, gas transmission, coal mining, power generation using coal, oil and gas, and coal gasification and liquefaction are among the fossil fuel based processes in the model. The operating levels of the processing units are limited in one way or the other by the available capacities and proven reserves in any period. The proven reserves may be augmented by the exploration activity. And, raw material imports/exports make up the difference between the domestic production and usage.

Among the linkages that interconnect the energy sector and the rest of the economy are (Exhibit 1): energy demands of the economy, total manpower available to all sectors (including energy) of the economy, favorable balance-of-payments requirement, and bill-of-goods needed for energy processing and capacity expansion.

In order to mitigate many of the distortions caused by price changes and inflation, the industrial process of the National Economy and the detailed energy sector will be represented in terms of physical flows. For the energy sector this is relatively easy to do because its activity can be treated in BTU terms. For the non-energy sector however, it is more difficult because (i) most industries produce a heterogeneous product thereby creating a need for developing a weighted index of the component physical outputs, and (ii) the input-output transactions are compiled in dollar terms, and money quantities depend on prices as well as physical flows. Moreover, the component prices unfortunately vary relative to one another over time, and so do the relative magnitudes of the component outputs. If these relative price and output variations among the components are assumed to be absent,[2] then a weighted index can be conveniently obtained by defining a composite product for the heterogeneous industry using base year prices as weights for base-year outputs. The dollar transactions are thence reinterpreted as physical units of the composit product.

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[2]Unless specific allowance is made, this assumption would be implcit in any temporal input-output model.

Whereas the input-output matrix represents the operating coefficients of the industrial processes of the economy, the capital coefficient matrix represents the amounts of the industrial products needed for a unit of (output) capacity expansion in any industrial (or energy) sector. Because a portion of the available capacity is retired at the end of each period, some capacity addition would be required in any scenario just to sustain the capacity of a process at a fixed level. This feature of the model also makes possible the process substitution. Thus, in the detailed energy sector where we expect to incorporate data of the new energy conversion technologies such as coal gasification and liquefaction, fuel cells, fast breeder reactors, etc., it will be possible to examine scenarios in which the distribution of capacities across the energy conversion processes evolves over time to reflect the impact of a particular set of assumptions specific to a scenario. On the other hand, in order to avoid the effort and difficulties involved in compiling reliable data of a similar nature for each of the other sectors of the economy, these will be initially represented by a nonvarying input-output matrix without substitution.

One of the primary linkages between the economy and the detailed energy sector is that of the energy sector meeting the demands of the economy. These energy demands are made up of the following four components: energy required for industrial processing, energy for personal (family) consumption, net exports of processed energy, and energy required to provide government services. In the model, these demands are transmitted to the energy sector in terms of the following four final energy forms:[3] oil products, gas products, coal and electricity. Moreover, this same set of demand variables is employed to compute the amounts of industrial goods and services required for energy extraction and processing.[4] The latter linkage also requires a modification of the input-output matrix.

The activities of the detailed energy sector are represented in two groups: nuclear and non-nuclear. The non-nuclear group contains for the most part the

[3] An alternative level of information detail would consist of eight final energy forms using the data developed by Knecht and Bullard [1975]. We may experiment with this form of the linkage at a later date.

[4] Using data similar to those developed by Just et al. [1975], it is possible to incorporate a full blown operating coefficient matrix to more accurately provide this linkage.

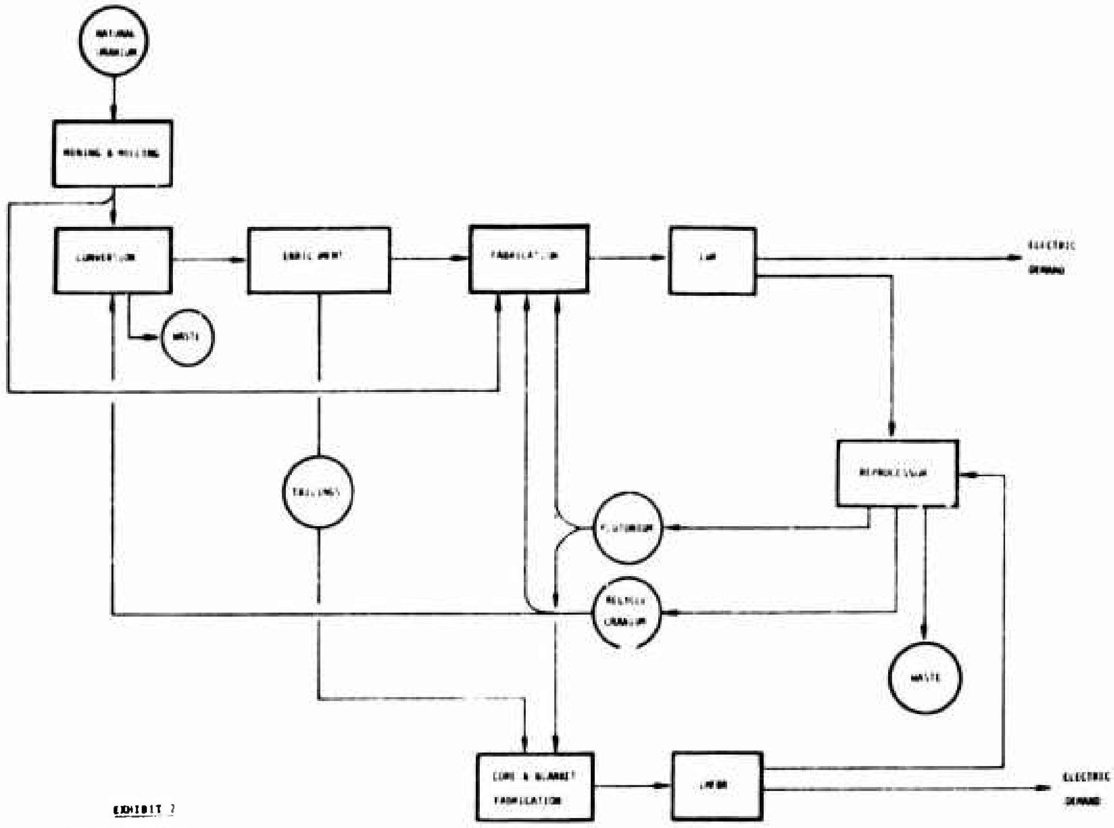


EXHIBIT 2

LWR AND LMFBR TECHNOLOGY

IN THE ENERGY SECTOR OF PILOT

(SUGGESTED BY T J CONNOLLY), 4/75

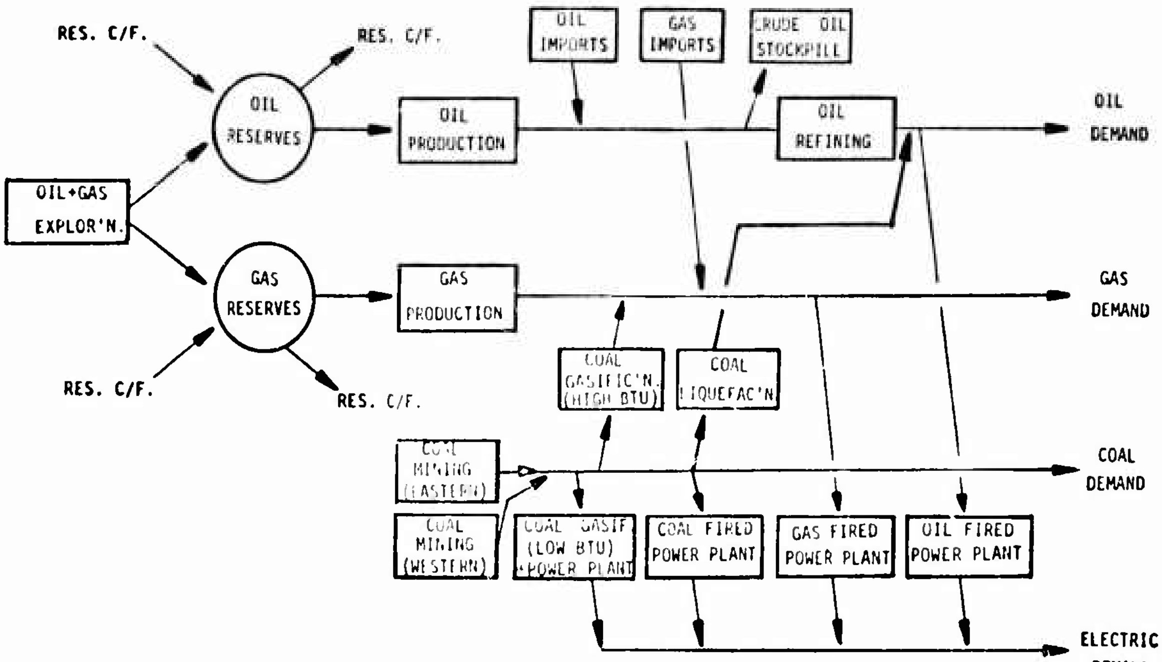


EXHIBIT 3

FOSSIL FUEL BASED ACTIVITIES

OF THE ENERGY SECTOR OF PILOT

fossil fuel based activities. It also includes the fuel free activities such as hydroelectric, geothermal, etc.

Exhibit 2 schematically shows the electric power generation related activities of the nuclear fuel cycle in the model. Natural or recycle uranium goes through chemical conversion and physical separation and enrichment before it is fabricated into the fuel elements for the light water reactor (LWR). Fuel elements could also be fabricated from recycle plutonium and uranium. The spent fuel may be reprocessed to recover the plutonium and uranium. The liquid metal fast breeder reactor (LMFBR) operation is similarly defined in the model.

Exhibit 3 shows the activities of the fossil fuel based energy processes in the model. Exploration for either oil or gas results in additions to the reserves of these raw energy forms. Oil and gas production, and coal mining activities provide the raw fossil fuels which are next processed into final energy forms. For oil, this involves a refining activity that produces some oil products (gasoline, heating oil, etc.) for satisfying final demands, and other oil products (residual fuel oil) for use in electric power generation. Due to nature of the linkage by which the energy sector meets the energy demands of the economy, the detailed yield structure of the refinery operations is not represented here. Natural gas is transmitted either to meet the final demands or for power generation. For coal, three alternative uses are defined in the model: to meet coal demands of the economy, for power generation, and for synthetic oil and gas.

One of the most important linkages in the model requires that all capacity building be constrained by the capacity of the economy to build capacity, either for capacity expansion or for replacement of the retired equipment. This set of constraints imparts a tendency in the model for a gradual evolution over time of the distribution of the capacities across the exploration, production, and conversion processes of the energy sector. Because, if too drastic a change in the capacity distribution were to occur, it would most likely drain an unusual amount of the economy's capability to build capacity and leave an insufficient ability to build capacity of the other industrial processes. On the other hand, the economy could expand at an unusually rapid pace its capacity expansion industries (construction, industrial machinery, etc.) in order to meet the need to speed up the changes in capacity distribution, but only at the expense of reduced ability to produce consumer goods thereby perhaps reducing the standard of living in the short run.

This descriptive model could be used in conjunction with a linear or a nonlinear objective. The objective could be a utility function measuring the standard of living achieved over time. It could also be to minimize dependence on foreign ore, or to maximize energy output, or to maximinze employment. Our intent is to develop on a pilot scale a reasonably accurate general description of the American Economy and a more detailed description of the energy sector in order to facilitate studies of the physical potential of the economy under (i) alternative objectives, (ii) changing availability of various forms of energy, (iii) changing desirability and economic feasibility of energy conversion technologies, etc.

The oil embargo of 1973-74 revealed a tip of the iceberg dealing with the reality that the changes in the energy picture not only can affect the short term standard of living by way of (mile) long waiting lines at the gas stations, but also may affect the long run standard of living through drastically increased prices. Such higher prices may reflect not only the political realities of the world's raw energy markets but also much increased physical effort on the part of the American Economy to provide from the domestic sources the energy needed to operate the economic machinery.

How will the standard of living be affected over time? Our first stab at incorporating the standard of living in the model is as follows. We define consumption profiles of families in various income levels. It is known, for example, that a family with low income spends less dollars on food but more of its dollar expenditure is spent on food relative to a family in a high income level. Whereas, a family with high income not only spends more on housing but also a large fraction of its dollar on housing. We expect to define about 5 to 7 such profiles. One possible objective function is that of maximizing the "gross national consumption" or equivalently "average per capita national consumption."

The purpose of having an objective function is to project a path for the economy that pushes against its capacities, i.e., not to project a depression economy. In examining the question of the objective, however, one is immediately faced with the prospect of finding a generally acceptable utility (or welfare) function for the entire country--a not too promising task, to say the least. A much more plausible approach is to incorporate some information on national welfare in the objective function and some in the constraints. The maximization of gross national consumption as defined by the income level profiles is one possibility. In any case, one thing seems certain. It will require a great deal of experimentation before a satisfactory objective function approach is realized.

3. General mathematical statement of the model. In the model, there are interperiod and intra period constraints (Exhibit 4). They are briefly outlined below. A somewhat more detailed description can be found in [5].

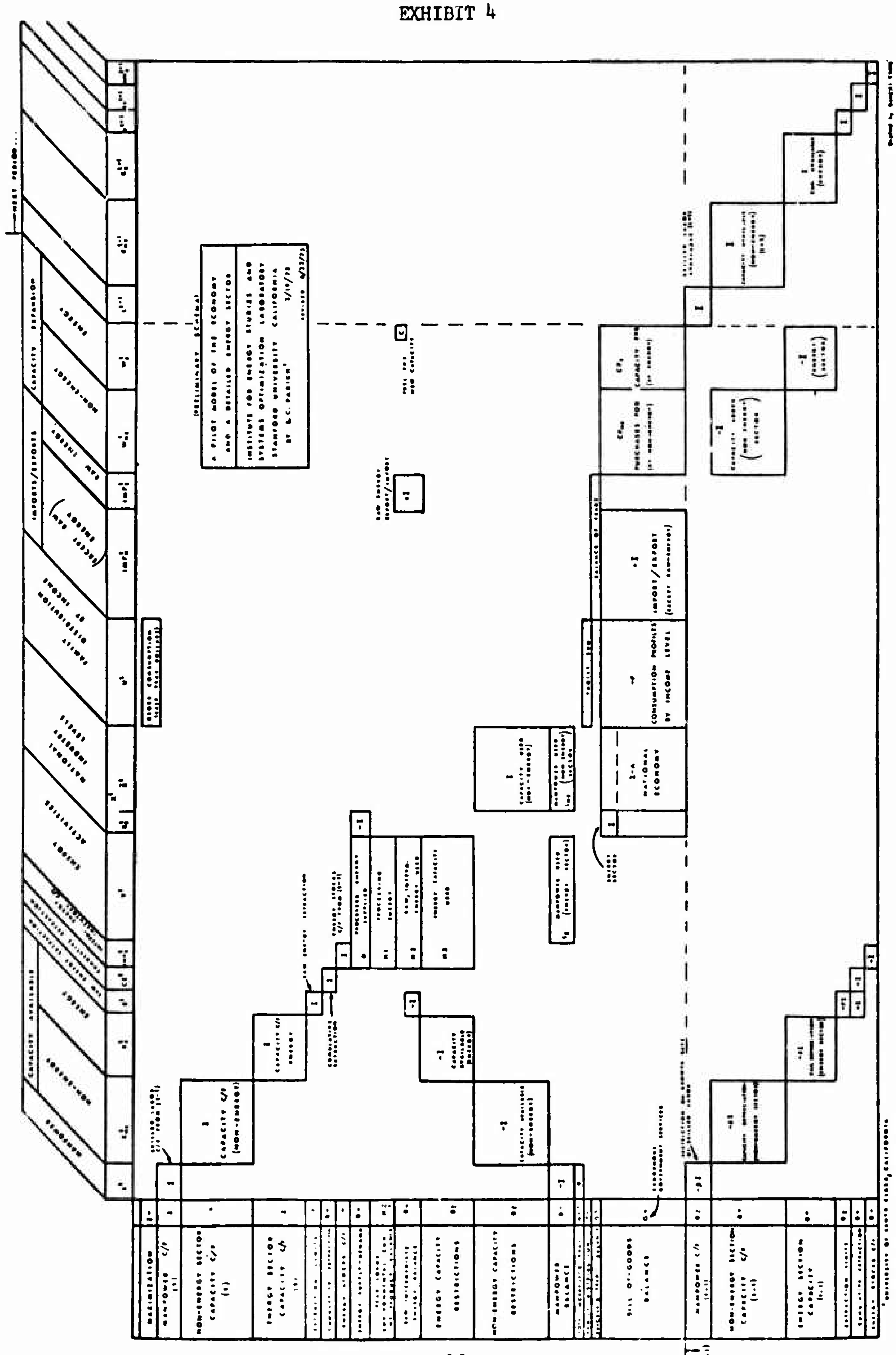
The interperiod constraints connecting periods t and (t+1) appear below the lower dashed line in Exhibit 4. These are capacity balance constraints, manpower skill adjustment limit constraints and those related to raw energy reserves, cumulative exploration and production, and intermediate energy stocks. The capacity balance (or capacity c/f) constraints specify that the available capacity in period (t+1) of any activity equals its capacity in period t, less retirements plus capacity built. Next, the manpower is assumed to be made up of several skill groups, e.g., unskilled, skilled, engineers, managers, etc. The manpower skill adjustment limit constraints (manpower c/f) specify the educational and training limitations. This set of constraints, together with the intraperiod constraint that the sum over all skill groups cannot exceed available workforce, provides for changes in the size of skill groups to satisfy the manpower needs.

The following 3 sets of interperiod constraints are intended to keep track in detail of the energy reserves, cumulative exploration (and production), and stocks. The reserves constraints specify that

$$\begin{bmatrix}\text{Reserves in}\\ \text{period } (t+1)\end{bmatrix} = \begin{bmatrix}\text{Reserves in}\\ \text{period } t\end{bmatrix} - \begin{bmatrix}\text{Raw energy}\\ \text{extracted}\\ \text{in period } t\end{bmatrix} + \begin{bmatrix}\text{Additions to}\\ \text{reserves in}\\ \text{period } t\end{bmatrix}$$

Cumulative exploration in say, feet drilled (and production in say, BTU's extracted) is determined as follows:

$$\begin{bmatrix}\text{Cumulative exploration}\\ \text{at the beginning of}\\ \text{period } (t+1)\end{bmatrix} = \begin{bmatrix}\text{Cumulative exploration}\\ \text{at the beginning of}\\ \text{period } t\end{bmatrix} + \begin{bmatrix}\text{Exploration}\\ \text{during}\\ \text{period } t\end{bmatrix}$$


A PILOT MODEL OF THE ECONOMY AND A DETAILED ENERGY SECTOR
INSTITUTE FOR ENERGY STUDIES AND SYSTEMS OPTIMIZATION LABORATORY STANFORD UNIVERSITY CALIFORNIA
CAPACITY AVAILABLE
CAPACITY EXPANSION
IMPORTS/EXPORTS
NATIONAL ECONOMY
CONSUMPTION PROFILES BY INCOME LEVEL
IMPORT/EXPORT
CAPACITY C/P (NON-ENERGY)
MAXIMIZATION
MANPOWER BALANCE
BILL OF GOODS BALANCE

The production of an energy form may be direct as in the case of oil, gas, uranium, etc., or a byproduct of some other acitivity, as in the case of plutonium. Finally, the stock (inventory) balance constraints: For ith energy form,

$$\begin{bmatrix}\text{Stocks at the}\\ \text{beginning of}\\ \text{period (t+1)}\end{bmatrix} = \begin{bmatrix}\text{Stocks at the}\\ \text{beginning of}\\ \text{period t}\end{bmatrix} + \begin{bmatrix}\text{amount produced}\\ \text{during}\\ \text{period t}\end{bmatrix} - \begin{bmatrix}\text{amount used}\\ \text{during}\\ \text{period t}\end{bmatrix}$$

Whether a constraint from these 3 sets is included in the model for a particular energy form depends upon its need and/or validity. For example, one may leave out the stock balance constraint for natural gas by arguing that gas could be extracted only if needed during the period. On the other hand, the exploration constraint for plutonium is invalid. The constraints from these sets in Exhibit 4 are for illustration only.

The intraperiod constraints appear between the two horizontal dashed lines in Exhibit 4. The first set (involving matrix block D) provide for meeting the energy demands of the economy from the energy sector. The next two sets (involving matrix blocks H1 and H2 represent various energy processing aspects. The environmental aspects of energy extraction and conversion could also be included here. The next two sets (involving matrix block H3, and variables x^t, K^t_{NE}) specify the operating capacity limitations of the energy and non-energy processes. The next set (involving L_{NE} and L_E) specify the manpower constraints that for each skill group, the manpower used cannot exceed that available. The next constraint states that manpower sum across skills cannot exceed available workforce. The family sum equation is used in conjunction with the objective function described below. The balance of trade equation computes the trade balance in each period for the purpose of incorporating a favorable trade balance requirement. Such a requirement may be imposed individually in each period or collectively in several periods. Finally, the bill-of-goods balance equations specify that the industrial output, together with imports (IMP) meets final demands consisting of personal consumption (Fu), exports (EXP), capacity expansion ($CP_{NE}W_{NE} + CP_EW_E$), and government expenditures (G).

Now consider the form of the objective function in the model. Broadly speaking, the objective of the model is to maximize the discounted vector bill-of-goods received per person summed over time. Suppose that in the base year, the physical bill-of-goods for people with consumption level M_k (income less taxes and savings in base year dollars) is: $b^k = [b_{1k}, b_{2k}, \ldots, b_{nk}]^T$, $k = 1, \ldots, K$ and with $M_1 < M_2 < \cdots < M_K$. Let u_k^t be the unknown number of people in period t that receive b^k. Then, $u_1^t + u_2^t + \cdots + u_K^t = P(t)$, the population at time t. The total bill-of-goods for period t is Fu^t, where: $F = [b^1, b^2, \ldots, b^K]$, and $u^t = [u_1^t, u_2^t, \ldots, u_K^t]^T$. Initially, the overall objective will be to maximize discounted gross national consumption over time, i.e., maximize $\Sigma_{t=1}^T \lambda_t \, GNC(t)$, where $GNC(t) = M_1 u_1^t + \cdots + M_K u_K^t$, and λ_t = weight in period t for discounting. It should be noted here with caution that the treatment of the objective function may change, even drastically, as experiments are performed on this model and numerical results become available. For a discussion of the use of production functions, demand functions and other forms of the objective, see [4] [4a].

Finally, unless specific allowance is made, an optimal solution to the model may turn out to be such that all (or most) capacity is depleted by the end of the time horizon T. Such unrealistic end effects can be avoided in several ways. One is to put a much higher weight λ_T on $GNC(T)$, the gross national consumption in the last period. Such a weight conceptually would reflect the present value of consumption beyond T. Another way is to specify the terminal capacities generated by an equilibrium model or a steady growth model. For the energy sector, this specification could be in gross BTU terms across several processes, thereby allowing for changes in capacity distribution across processes.

4. Further remarks. The model formulation and data source identification are almost complete. Presently, the data are being aggregated and a model is being prepared for computer solution. We expect the model to have about 125 equations per period. For a 30-year triannual model, there will be about 1250-1400 equations, including the specification of initial capacities and end effects. Initially, the model will be solved using the straight simplex method of the MPS/370 system.

In order to pave a way for economical solution of similar much larger problems having 8,000-10,000 rows referred to earlier, it is also expected that the PILOT model will provide us at the Systems Optimization Laboratory a prototype for research in solving large-scale linear programming models of energy systems.

The PILOT model belongs to a class of models having a staircase structure (Exhibit 5) that often arises in dynamic linear programs. For such time-phased problems, the number of iterations to optimum may be as high as 10 times the number of rows as opposed to widely experienced 2 to 4 times the row count in unstructured problems [Beale, 1971].

Several special purpose algorithms are available that take advantage of the staircase structure for efficient solution. Computational results on some of the methods show that this is a partly-proven and a very promising research area. See, for example, Dantzig [1963, Ch. 23], Glassey [1973], and Ho [1974], just to mention a few.

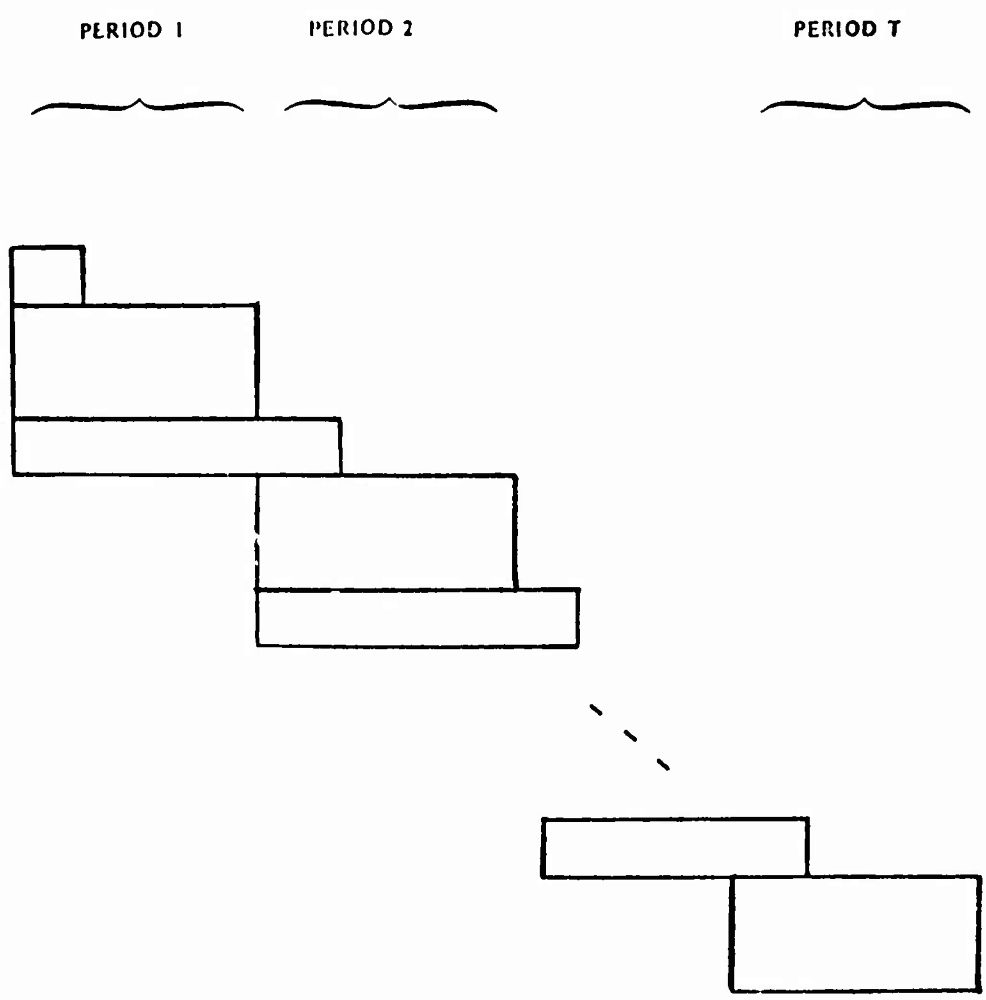


EXHIBIT 5. THE STAIRCASE STRUCTURE OF PILOT

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